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(2) $\int \cos^n x \, dx$

Let $I_n = \int \cos^n x \, dx$

where n is positive Integer.

$$I_n = \int \cos^{n-1} x \, (\cos x) \, dx$$

Applying the method of integration by parts, we get

$$I_n = \cos^{n-1} x \, (\sin x) - \int (n-1) \cos^{n-2} x \, (-\sin x) \sin x \, dx$$

$$= \cos^{n-1} x \, \{ \sin x + (n-1) \int \cos^{n-2} x \sin^2 x \, dx$$

$$= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \, (1 - \cos^2 x) \, dx.$$

$$= \cos^{m-1} x \sin x + (n-1) \int (\cos^{n-2} x - \cos^n x) \, dx$$

$$= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x - (n-1) \cdot \int \cos^n x \, dx$$

$$I_n = \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n$$

$$I_n + (n-1) I_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

$$I_n (n - \cancel{1} + \cancel{1}) = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

$$n I_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

$$I_n = \frac{1}{n} \cos^{n-1} x \sin x + \frac{(n-1)}{n} I_{n-2}$$

This is the reduction formula for $\int \cos^n x \, dx$